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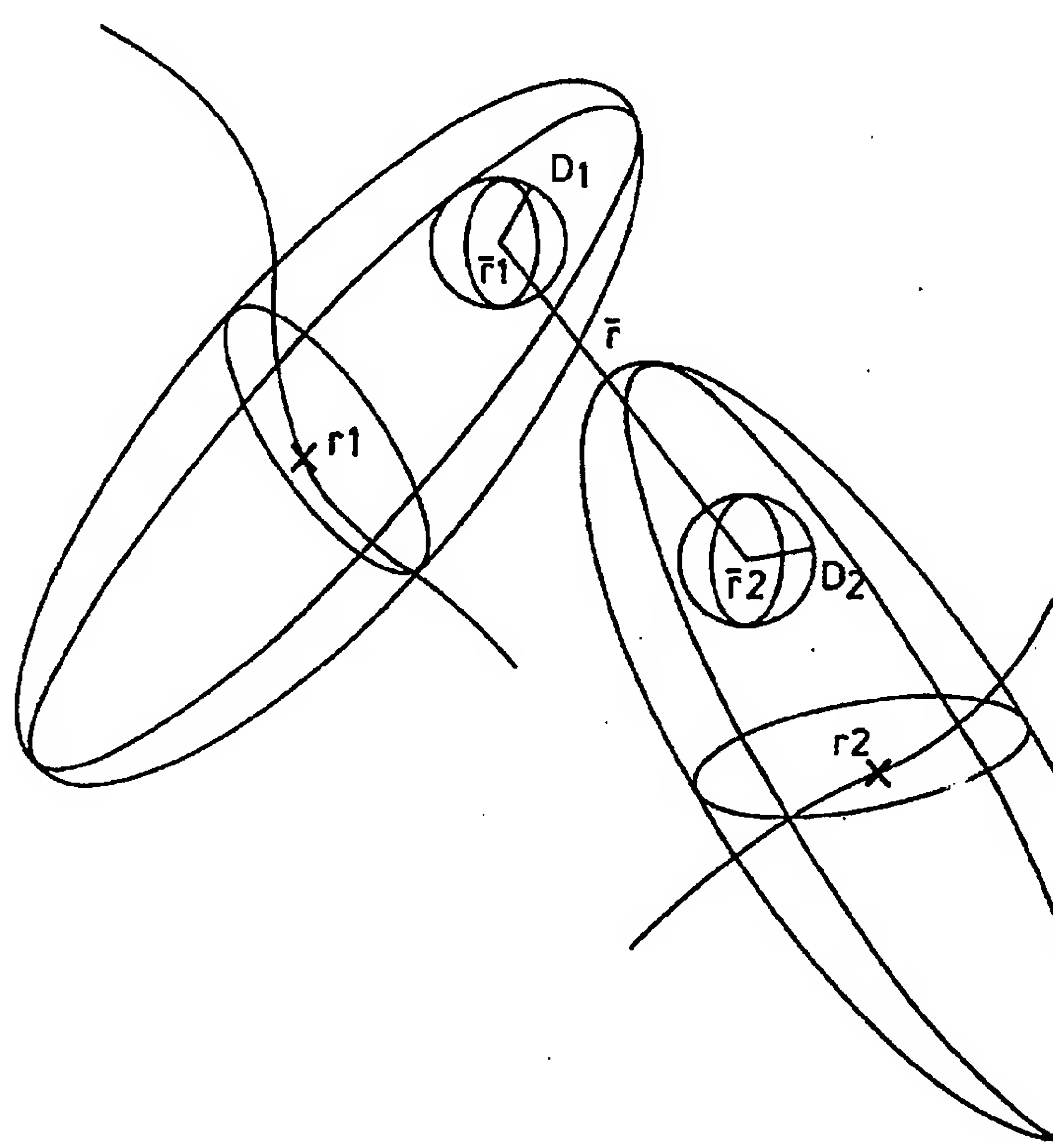
(57) Abstract

A method is disclosed of determining position uncertainty of directional boreholes A, B in which the true position of well A, $\bar{r}^1$, will lie in the ellipsoid centred about r1, and the true position of well B, $\bar{r}^2$, will lie within the ellipsoid centred around r2. The method comprises measuring either the azimuth and inclination of the borehole at intervals along it, or the total displacement from the surface location of successive points, determining the magnitude of systematic and random errors inherent in the measurements, determining the variance matrix describing the position uncertainty of each station in the survey, determining the point of closest approach of the borehole to a point of interest, determining the variance matrices representing the position uncertainty of each of the point of interest and the point of closest approach, determining the vector between the point of interest and the point of closest approach, determining the normalised distribution of the length of the vector according to the variance matrices, and integrating the normalised distribution to obtain the probability that the borehole lies within the specified radius of the point of interest.



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A PROCESS FOR DIRECTIONAL DRILLING

The present invention relates to methods of determining the positional uncertainty of a directional borehole, and relates particularly , but not exclusively, to methods of determining positional uncertainty of a borehole of an oil or gas well.

Controlled directional drilling is the process of maintaining the trajectory of a borehole along a predefined course from a surface location to one or more subsurface target locations. Directionally drilled boreholes are widely used for the exploitation of subsurface natural resources such as oil and gas, and coal bed methane, and are also used during the construction of so-called "freeze rings" whilst sinking mine shafts through water bearing formations, for scientific purposes including the analysis of ground water flow and the performance and evaluation of underground nuclear tests, as well as for the controlled extraction of ground water contaminated with environmentally damaging chemicals and the creation of conduits for pipes or cables beneath rivers, lakes or other obstacles.

A variety of techniques may be employed to deviate the path of a directionally drilled borehole. When rotary drilling, buckling of drill collars within the bottom section of drill string (the "bottom hole assembly (BHA)") can be controlled through careful selection of the position and gauge of stabilisers, and thus the tendency of the BHA to increase, maintain or decrease its angle from the vertical may be changed. The stabilisers employed may be of fixed gauge or have gauge adjustable from the surface, and in either case control over inclination is more readily achieved than direction. Downhole motors are commonly used to achieve better control over direction, and power the drill bit so that the drill string need not be rotated. By introducing a "bent sub" into the BHA above the motor it may be orientated with respect to the axis of the borehole and thus induced to drill a smooth arc.

Controlled directional drilling requires that the path of the borehole is surveyed during drilling in order that appropriate steps may be taken to maintain the well path along a desired trajectory. It is extremely difficult precisely to control the buckling or orientation of a BHA and thus accurately drill in a predetermined direction. Similarly, measurements of the borehole vector, i.e. couplets of measurements of borehole inclination from vertical and azimuth (the observed borehole direction measured from north) are subject to errors in inclination, azimuth, and along-hole depth. Computation of position along the borehole thus leads to an accumulation of error so that uncertainty in the position of stations (i.e. points at which observations of borehole vector are made) is greater in stations further along the borehole, and lesser in stations closer to the origin.

Borehole position uncertainty causes considerable problems for applications of controlled directional drilling. In particular, the economic development of oil and gas fields off shore, and in the Arctic and other environmentally sensitive areas, frequently calls for a controlled directional drilling of multiple wells radiating from single platform, sub-sea template or rig site. In such circumstances, wells are closely spaced at the surface, and there is a serious risk that any new well may intersect an existing well, any such collision risking the release of hydrocarbons into the environment, and subsequent fire and pollution. The actions required to minimise the impact of such a collision include "shutting in" the well, i.e. ceasing production or injection and eliminating excess pressure within the borehole, and adopting more cautious and time consuming drilling methods. Both of these options are extremely costly.

A further problem of borehole position uncertainty is encountered when attempting to deliberately intersect an existing borehole. Such action may be necessary if control of an existing producing well is lost in a "blow out", and has also proved necessary when re-entering an abandoned well in order to undertake fresh operations. In both cases, lack of

knowledge of the borehole position requires that expensive "well location" logging equipment be run.

Computer models have been used for a number of years to predict borehole position uncertainty and the results used to reduce the risk of well collision by computing the separation of boreholes from one another and accounting for their positional uncertainties in an empirical manner. Such methods have been introduced to systems of control employed at rig sites to provide regular "go / no-go" decisions during the critical section down to and immediately below the "kick-off".

Such borehole uncertainty models tend to be variations on models first published by Walstrom J E, Brown A A and Harvey R P - An analysis of Uncertainty in Directional Surveying - Journal of Petroleum Technology, April 1969, PP515-522 or, more commonly, Wolff C J M and de Wardt J P - Borehole Position Uncertainty, Analysis of Measuring Methods and Derivation of Systematic Error Model - Journal of Petroleum Technology, December 1981, PP2339-2350. Such methods may be used to develop ellipsoids bounding the volume within which the borehole location may be expected to lie.

The primary method of borehole intersection avoidance currently used is to generate such ellipsoids or ellipses at regular intervals along drilled and proposed boreholes and to compare the separation of the boreholes to the size and position of the ellipses. This comparison may be undertaken visually, or through identification of the point of closest approach of the proposed borehole to the drilled borehole and subsequent computation of a simple ratio known as the separation or clearance factor defined by $F = R / (r_1 + r_2)$, where r_1 and r_2 are the radii of the ellipses of uncertainty on the proposed and drilled boreholes respectively along the direction between them and R is the separation along that same direction.

Borehole collisions are most likely to occur where separation is low, which is generally near the surface. Since there the boreholes are nearly vertical and their ellipses of uncertainty approximately circular, a factor F greater than 1

implies that the two ellipses of uncertainty do not overlap one another and if, as in the method of Wolff and de Wardt above, the ellipses are held to bound all possible positions of the borehole location, then collision of the two boreholes cannot occur. At higher borehole inclinations, the ellipses are no longer approximately circular, and a separation factor greater than 1 no longer implies that the ellipses cannot intersect. In such circumstances, r_1 and r_2 may be replaced by the major axes of the two ellipses to produce a "fail safe" solution.

Typically, criteria are set down defining actions to be taken during drilling at values of F approaching 1. For example, in order to continue drilling at F = 1.5, it may be necessary to shut in and bleed off excess pressure in an adjacent well. At F = 1.25, surveys of the borehole being drilled may be required to be taken more frequently or with higher precision sensors. When F is less than 1, it may be deemed necessary to cease drilling, plug the borehole and deviate it towards a safer area.

All of the above practises suffer from the disadvantage of considerable expense in terms of time, equipment, and deferred production from the well. In particular, prior art techniques suffer from the disadvantage that they do not permit a quantative estimation of the risk of borehole intersection, and can therefore cause safety procedures to be prematurely invoked such that substantially more cost is incurred than the actual risk justifies.

This inability quantitatively to estimate the risk of borehole intersection has substantial cost implications. The slot separation on off shore structures and sub-sea templates is typically of the order of 10 feet, largely to ensure that the ellipses of uncertainty of the boreholes drilled therefrom do not intersect. For many off shore structures, especially those to be drilled from jack-up drilling units, the topside weight is proportional to the area, which is in turn related to the square of the slot spacing. Likewise, the jacket weight is proportional to the top side weight, and the ability to reduce slot separation without compromising safety would be highly

desirable in terms of the substantial capital savings gained.

Preferred embodiments of the present invention seek to overcome the above disadvantages of the prior art.

According to an aspect of the present invention, there is provided a method of determining position uncertainty of a directional borehole, the method comprising:

measuring azimuth and inclination of the borehole at intervals along it using magnetic or gyroscopic instruments, or measuring the total displacement from a surface location of successive points using an inertial instrument;

determining the magnitude of systematic and random errors inherent in the measurements made and their relationship with azimuth and inclination of the borehole and displacement from the origin and elapsed time at each successive point of observation;

determining a first variance matrix representing position uncertainty at each location at which said measurements are made;

determining the point of closest approach of the borehole to a reference point;

determining second variance matrices representing the position uncertainty at said reference point and said point of closest approach;

determining a vector between the reference point and the point of closest approach;

determining a normalised distribution of the length of said vector according to the second variance matrices; and

integrating the normalised distribution between limits of zero and a value equal to the sum of a predetermined radius and the radius of the borehole, wherein the integral obtained represents the probability that the borehole lies within the predetermined radius of the reference point.

This provides the advantage of permitting a quantative assessment of the risk of borehole intersection to be made which in turn enables borehole separation to be reduced without compromising safety.

In a preferred embodiment, the step of determining the

first and / or second variance matrix and / or the normalised distribution are performed by application of a stochastic technique, and preferably by application of Monte Carlo modelling, and the integrating step performed by examination of the frequencies thus derived.

In a preferred embodiment, the first variance matrix is determined from estimates of expected drilling precision.

Preferably, the reference point lies along an existing or intended borehole.

The integrating step preferably further comprises setting the predetermined radius around the reference point to the radius of the borehole along which the reference point lies.

This provides the advantage that the integral obtained then defines the probability of intersection of two boreholes.

The method is preferably applied iteratively to a set of boreholes and the integrals obtained from the integrating step are summed.

In this way, the summation defines the probability of any of the boreholes being within a specified radius of the reference point.

The method may further comprise the step of geometrically transforming the second variance matrices, prior to the determination of the normalised distribution, into an arbitrary co-ordinate system and said integrating step comprises restricting the integral to a single dimension so that it represents the probability that the borehole lies within a specific distance and direction of the reference point.

In a preferred embodiment, the reference point is the position of the bit of a borehole drilling apparatus located within a borehole, and the arbitrary co-ordinate system is defined by the direction and inclination of the borehole at that position.

This provides the advantage that the integral determined in the integrating step defines probabilities in either of the directions ahead / behind, left / right or above / below the bit.

According to another aspect of the present invention,

there is provided a method of borehole drilling comprising a method as defined above, and controlling drilling of the borehole according to the probability obtained from the integrating step.

According to a further aspect of the present invention, there is provided a method of determining position uncertainty in a directional borehole, the method comprising:

measuring the azimuth and inclination of the borehole at intervals along it using magnetic or gyroscopic instruments, or measuring the total displacement of successive points from a surface location using an inertial instrument;

determining the magnitude of systematic and random errors inherent in the measurements made;

modelling the relationship of the magnitude of both systematic and random errors with azimuth and inclination at the measuring locations, and its displacement from the origin and elapsed time as a weighting function;

determining a variance matrix representing position uncertainty of each measurement location; and

obtaining from the variance matrix an ellipsoid bounding the volume of space containing the probable position at a predetermined level of confidence of each measurement position.

In a preferred embodiment, the step of determining the variance matrix accounts for the contribution to the variance matrix of the uncertainty in the position of the surface location.

Preferably, the step of determining the variance matrix accounts for the contribution to the variance matrix of the correlation between individual systematic error terms in successive sections of a borehole survey run in multiple sections.

According to a further aspect of the present invention, there is provided a method of determining position uncertainty in a directional borehole, the method comprising:

measuring the azimuth and inclination of the borehole at intervals along it using magnetic or gyroscopic instruments, or measuring the total displacement of successive points from a

surface location using an inertial instrument;

determining the magnitude of systematic and random errors inherent in the measurements made;

modelling the relationship of the magnitude of both systematic and random errors with azimuth and inclination at the measuring locations, and its displacement from the origin and elapsed time as a waiting function;

determining a variance matrix representing position uncertainty of each measurement location; and

obtaining from the variance matrix an ellipse defining an area containing the probable position at a predetermined level of confidence of each measurement position, or obtaining the probability in a predetermined direction.

The method may further comprise the step of determining the minimum number of survey sections and measurement locations required to achieve a predetermined level of confidence by iterative application of the method defined above.

A preferred embodiment of the invention will now be described, by way of example only and not in any limitative sense, with reference to the accompanying drawings in which:-

Figure 1 is a schematic illustration of two boreholes and ellipsoids of position uncertainty surrounding points on the boreholes; and

Figure 2 is a schematic illustration of the probability of collision between a pair of directional boreholes.

Nomenclature

V	The variance matrix at a particular station
COV	The covariance matrix at a particular station.
H	The matrix used to represent the covariance matrix.
N, E, V	North, East and Vertical respectively
var (,)	The variance of positional uncertainties.
x, y, z	x, y, z coordinates
cov(,)	The covariance positional

uncertainties.

Δr The position vector resulting from the measuring errors.

Δm_j The magnitude of the individual j^{th} systematic measuring error.

ΔM_j The maximum magnitude of the individual j^{th} systematic measuring error.

$\overline{\Delta m_j}^{(i)}(s)$ The magnitude of the j^{th} random measuring error at the i^{th} station.

$w_j(s)$ The weighting function that takes into account that individual measuring errors may depend upon the actual borehole vector or duration of the survey.

$v_j(s)$ The unit vector indicating the direction in which the measuring error is effective.

D_i The measured along hole depth at the i^{th} station.

$\Delta m_j^{(i)}(s)$ The magnitude of the individual j^{th} measuring error for the i^{th} instrument used in a survey comprising consecutive runs using different survey instruments.

D The total along hole depth of the survey obtained using a specific instrument in a survey comprising consecutive runs using different survey instruments.

D_{AH} The total along hole depth of the borehole.

r The expected position of a particular station.

$\bar{r}$ The true Position of a particular station.

Δr The error vector.

d The minimum distance between two

stations.

R	The borehole radius.
σ_x^2	The variance of the error distribution of the x direction.
σ_{xy}	The covariance between the error distributions in the x direction and the y direction.
ρ_{xy}	The correlation coefficient given by $\dfrac{\sigma_{xy}}{\sqrt{\sigma_x^2}\ \sqrt{\sigma_y^2}}$
Δm_j	The maximum value of the j^{th} individual measuring error.
λ	The parameter determining the size of the ellipsoid.
λ'	The parameter used to scale the individual measuring errors.
P	The probability.

The meanings of a_j, $a_j^{(i)}$, $a_j^{(1)}$, and $a_j^{(2)}$ are described in the text.

When a borehole is surveyed, imprecision in the observations leads to the development of a degree of uncertainty in the computed position at each point (or station) of the survey. This uncertainty is typically expressed as a variance matrix, the elements of which define the variances and covariances of the error in position in the North, East and Vertical directions. To facilitate visualisation of the position uncertainty, ellipsoids of the form

$$\Delta r^T V^{-1} \Delta r = \lambda^2$$

where V is the variance matrix and Δr is the position vector resulting from the error vectors, which describe volumes of equal probability density, may be extracted from the variance matrix.

The basic objective of the improved method is to determine the positional uncertainty of each station in a borehole survey. This may be represented by either an ellipsoidal volume of uncertainty centred about the expected

position of the station or by a covariance matrix. The covariance matrix at a particular station is determined by taking the expected value of the matrix formed by premultiplying the column vector of the positional error about the expected value of the error at that station with its row vector (i.e. its transpose). In other words:

$$COV = E\ \{[\Delta r - E\ (\Delta r)]\,[\Delta r - E\ (\Delta r)]^T\}$$

This matrix contains a statistical description of the probability distribution of the position vector of the station under consideration and has dimensions given by:

$$COV = \begin{bmatrix} \mathrm{var}(\Delta E, \Delta E) & \mathrm{cov}(\Delta E, \Delta N) & \mathrm{cov}(\Delta E, \Delta V) \\ \mathrm{cov}(\Delta E, \Delta N) & \mathrm{var}(\Delta N, \Delta N) & \mathrm{cov}(\Delta N, \Delta V) \\ \mathrm{cov}(\Delta E, \Delta V) & \mathrm{cov}(\Delta N, \Delta V) & \mathrm{var}(\Delta V, \Delta V) \end{bmatrix}$$

The eigenvalues and eigenvectors of the covariance matrix determine the axes of an ellipsoid containing the possible positions of the station when all the individual measuring errors are taken into account.

In the analysis below it is assumed that the probability distribution of each source of measurement error is bounded by finite upper and lower limits. This assumption implies that the ellipsoid will contain all the positions that are possible for the station.

If, on the other hand, it is assumed that the distribution of each error is not bounded then, by the Central Limited Theorem, the ellipsoid describes the volume containing a three-variate normal distribution (i.e. normal distributions in each of East, North and Vertical directions) of possible station locations.

Present art leans towards extracting the ellipsoid from the covariance matrix at an early stage, thereafter manipulating it as a geometrical volume rather than a

statistical entity. This approach has developed because of the difficulty of visualising the abstractions embodied in the covariance matrix and has led to a number of difficulties. It is meaningless, for example, to combine error ellipsoids as summation of geometric characteristics such as the length of axes or the enclosed volume does not reflect the correct manner of combining variances of covariances. It is, therefore, desirable to retain the covariance matrix for as long as possible; this approach is taken in the methods described below.

Systematic and random measurements errors may be combined to produce a covariance matrix which fully describes the positional uncertainty of a station.

If an East, North, Vertical co-ordinate system is assumed the positional uncertainty Δr at the first station is given by:

$$\mathbf{\Delta r} = \sum_j \Delta m_j \int_0^{D_1} w_j(s)\, v_j(s)\, ds + \sum_j \Delta\tilde{m}_j^{(1)} \int_0^{D_1} w_j(s)\, v_j(s)\, ds$$

where the first term on the RHS represents the contribution to the positional uncertainty of the systematic error terms and the second term represents the contribution of the random error terms.

The individual systematic error terms, Δm_j, are considered to be the same at each station. Their actual values for any specific survey are unknown, but are drawn randomly from distributions centred on their expected values. The values for the individual random error terms, $\Delta\tilde{m}_j^{(1)}$, vary from station to station; these too are drawn randomly from distributions centred on their expected values.

Since, the systematic error terms at the second station are the same as the first, the positional uncertainty of the second station will be:

$$\Delta r = \sum_j \Delta m_j \int_0^{D_2} w_j(s)\, v_j(s)\, ds + \sum_j \Delta \bar{m}_j^{(1)} \int_0^{D_1} w_j(s)\, v_j(s)\, ds + \sum_j \Delta \bar{m}_j^{(2)} \int_{D_1}^{D_2} w_j(s)\, v_j(s)\, ds$$

Hence, it follows that the positional uncertainty at the nth station is:

$$\Delta r = \sum_j \Delta m_j \int_0^{D_n} w_j(s)\, v_j(s)\, ds + \sum_j \Delta \bar{m}_j^{(1)} \int_0^{D_1} w_j(s)\, v_j(s)\, ds + \sum_j \Delta \bar{m}_j^{(2)} \int_{D_1}^{D_2} w_j(s)\, v_j(s)\, ds + \ldots + \sum_j \Delta \bar{m}_j^{(n)} \int_{D_{n-1}}^{D_n} w_j(s)\, v_j(s)\, ds \qquad (1)$$

As mentioned above the covariance matrix at a station with a position vector, r, has the form:

$$COV = E\ \{[\Delta r - E(\Delta r)]\ [\Delta r - E(\Delta r)]^T\}$$

This may be expanded to give:

$$COV = E\ \{\Delta r \Delta r^T\} - E\ \{\Delta r\}\ E\ \{\Delta r^T\}$$

This represents a matrix of the form $E\ \{\Delta r \Delta r^T\}$ whose centre is about the position vector of the station, r, plus the expected value of the positional error $E\{\Delta r\}$ at r.

Thus, writing:

$$a_j = \int_0^{D_n} w_j(s)\, v_j(s)\, ds \quad \text{and} \quad a_j^{(i)} = \int_{D_{i-1}}^{D_i} w_j(s)\, v_j(s)\, ds$$

the covariance matrix:

$$COV = E\{\Delta r \Delta r^T\}$$

on substituting (1) becomes:

$$COV = E\left\{\left(\Sigma_j \Delta m_j a_j + \Sigma_j \Delta \tilde{m}_j^{(1)} a_j^{(1)} + \ldots + \Sigma_j \Delta \tilde{m}_j^{(n)} a_j^{(n)}\right)\left(\Sigma_j \Delta m_j a_j + \Sigma_j \Delta \tilde{m}_j^{(1)} a_j^{(1)} + \ldots + \Sigma_j \Delta \tilde{m}_j^{(n)} a_j^{(n)}\right)^T\right\}$$

which gives:

$$\begin{aligned} COV = {} & \Sigma_j \Sigma_k E(\Delta m_j \Delta m_k) a_j a_k^T + \Sigma_j \Sigma_k E\left(\Delta m_j \Delta \tilde{m}_k^{(1)}\right) a_j a_k^{(1)T} + \ldots \\ & + \Sigma_j \Sigma_k E\left(\Delta \tilde{m}_j^{(1)} \Delta m_k\right) a_j^{(1)} a_k^T + \ldots \\ & + \\ & \cdot \\ & \cdot \\ & \cdot \\ & + \Sigma_j \Sigma_k E\left(\Delta \tilde{m}_j^{(n)} \Delta m_k\right) a_j^{(n)} a_k^T + \ldots + \Sigma_j \Sigma_k E\left(\Delta \tilde{m}_j^{(n)} \Delta \tilde{m}_k^{(n)}\right) a_j^{(n)} a_k^{(n)T} \end{aligned}$$

Since the error terms $\Delta\bar{m}_j^{(1)}$. $\Delta\bar{m}_j^{(2)}$ ..., $\Delta\bar{m}_j^{(n)}$ are random, these terms must be uncorrelated with each other. If it is also assumed that the individual systematic error terms are uncorrelated both with each other and with the random terms, it follows that the covariance matrix will be:

$$COV = \Sigma_j a_j a_j^T E\left(\Delta m_j^2\right) + \Sigma_j a_j^{(1)} a_j^{(1)T} E\left(\Delta \tilde{m}_j^{(1)2}\right) + \ldots + \Sigma_j a_j^{(n)} a_j^{(n)T} E\left(\Delta \tilde{m}_j^{(n)2}\right) \quad (2)$$

The above assumption is valid provided no individual measurement of a tool is dependent on any other individual measurement.

Equation (2) represents the covariance matrix formed by summing the systematic errors together over the entire length of the borehole plus the sum of the covariances matrices formed by summing the random errors at each station. The position error at any station is, therefore, dependent not only upon the position vector of the station, (cf. Wolff and de Wardt) but also upon the number of stations before it in the traverse (cf. Walstrom et al.). The combination of these hitherto incompatible approaches is at significant improvement over the prior art.

As the number of stations in a survey increases, so there is a greater tendency for random measurement errors to cancel one another; the total contribution of random measurement errors to the covariance matrix therefore tends to a limiting value. This has implications for the planning of borehole surveys in that both the optimum station interval and, as will be seen from the next section, the optimum number of survey sections must be considered.

There are also implications for borehole surveying practices at shallow depths which remained unexposed by present art. Current, Wolff and de Wardt based, models not only underestimate uncertainty because the contribution of random errors is not considered, but also, as the number of stations is low, are in error by a changing degree.

It is a common practice to survey boreholes in multiple sections which are then concatenated ("tied") into one survey. The individual surveys may be taken with tools of the same type (but perhaps not exactly the same instrument), but frequently different types of instrument are used in the different sections. It is now shown how the results of two or more such survey sections may be tied together. The technique may also be used to account for uncertainty in the surface location of the borehole.

Consider first the case where the survey comprises the results of only two tools, tied together at a "tie point" of depth D. In this case, the position error vector at a station at depth D_{AH} is given by:

$$\Delta \mathbf{r} = \Sigma_j \Delta m_j^{(1)} \int_0^D \mathbf{w}_j(s)\, \mathbf{v}_j(s)\, ds + \Sigma_j \Delta m_j^{(2)} \int_D^{D_{AH}} \mathbf{w}_j(s)\, \mathbf{v}_j(s)\, ds$$

+ random terms

where the first term on the RHS of the equation represents the contribution from the first tool and the second term represents the contribution from the second tool. The superscripts $^{(1)}$ and $^{(2)}$ refer to the first and second tools respectively.

If this is rewritten as:

$$\Delta r = \Sigma_j \Delta m_j^{(1)} a_j^{(1)} + \Sigma_j \Delta m_j^{(2)} a_j^{(2)} + \text{the random terms}$$

the covariance matrix:

$$COV = E\{\Delta r \Delta r^T\}$$

becomes:

$$COV = E\{\Sigma_j \Sigma_k \mathbf{a}_j^{(1)} \mathbf{a}_k^{(1)T} \Delta m_j^{(1)} \Delta m_k^{(1)}$$

$$+ \Sigma_j \Sigma_k \mathbf{a}_j^{(1)} \mathbf{a}_k^{(2)T} \Delta m_j^{(1)} \Delta m_k^{(2)} + \Sigma_j \Sigma_k \mathbf{a}_j^{(2)} \mathbf{a}_k^{(1)T} \Delta m_j^{(2)} \Delta m_k^{(1)}$$

$$+ \Sigma_j \Sigma_k \mathbf{a}_j^{(2)} \mathbf{a}_k^{(2)T} \Delta m_j^{(2)} \Delta m_k^{(2)}\}$$

+ the contribution of the random errors to COV

since the individual random errors are uncorrelated both with each other and the systematic errors.

It can be seen that the contribution to COV of the systematic errors is different depending on the assumptions made concerning the error data. If, as in the previous section, it is supposed that the individual sources of

measuring error for each tool are uncorrelated and ignoring, for the purposes of clarity only, the contribution of the random errors, the covariance matrix will become:

$$\begin{aligned}COV = {} & \Sigma_j a_j^{(1)} a_k^{(1)T} E\left(\Delta m^{(1)^2}\right) + \Sigma_j a_j^{(1)} a_k^{(2)T} E\left(\Delta m_j^{(1)} \Delta m_j^{(2)}\right) \\ & + \Sigma_j a_j^{(2)} a_k^{(1)T} E\left(\Delta m_j^{(2)} \Delta m_j^{(1)}\right) + \Sigma_j a_j^{(2)} a_k^{(2)T} E\left(\Delta m_j^{(2)} \Delta m_j^{(2)}\right)\end{aligned} \tag{3}$$

If, in addition, it is supposed that each source of measuring error in the first tool is uncorrelated with the corresponding error of the second tool, then the non-leading terms become zero and COV becomes:

$$COV = \Sigma_j a_j^{(1)} a_j^{(1)T} E\left(\Delta m^{(1)^2}\right) + \Sigma_j a_j^{(2)} a_k^{(2)T} E\left(\Delta m_j^{(2)^2}\right)$$

Extending equation (3) to consider a survey comprising n tied traverses, the equivalent form is:

$$\begin{aligned}COV = {} & \Sigma_j a_j^{(1)} a_j^{(1)T} E\left(\Delta m^{(1)^2}\right) + \Sigma_j a_j^{(1)} a_j^{(2)T} E\left(\Delta m_j^{(1)} \Delta m_j^{(2)}\right) + \ldots + \Sigma_j a_j^{(1)} a_k^{(n)T} E\left(\Delta m_j^{(1)} \Delta m_j^{(n)}\right) \\ & + \Sigma_j a_j^{(2)} a_j^{(1)T} E\left(\Delta m_j^{(2)} \Delta m_j^{(1)}\right) + \ldots \\ & + \ldots \\ & + \Sigma_j a_j^{(n)} a_j^{(1)T} E\left(\Delta m_j^{(n)} \Delta m_j^{(1)}\right) + \ldots \qquad + \Sigma_j a_j^{(n)} a_k^{(n)T} E\left(\Delta m_j^{(n)^2}\right)\end{aligned} \tag{4}$$

Note that the ratio between the number of non-leading terms to the number of leading terms, $\Sigma_j a_j^{(p)} a_j^{(p)T} E\left(\Delta m_j^{(p)2}\right)$ where $1 < p < n$, increases as n increases. This implies that the discrepancy between the correlated and uncorrelated forms of the covariance matrix will increase as n increases and hence it is important to establish the relationship between the error terms of the different tools when the covariance matrix for a tied survey is computed. Note also that, since the magnitude of the vector $a_j^{(p)}$ is dependent upon the length of the section,

p, so too is its contribution to the covariance matrix.

Also seen in (4) is an implication for the design of borehole surveys which is not exposed by present art, namely that by careful selection of the type of survey instrumentation such that the uncorrelated form of the matrix is appropriate, then dividing the borehole into a number of length sections, each making approximately the same contribution to the covariance matrix, will decrease the total error at depth.

Ellipsoids (or in two dimensions ellipses) of equal probability density can be denoted in vector notation by:

$$\Delta r^T COV \Delta r = \lambda^2$$

According to the method of Wolff and de Wardt hitherto employed a similar relationship exists between the variance, σ^2_j, and the quoted possible maximum error magnitudes, ΔM_j, of each measurement error

$$(\Delta M_j)^2 = \lambda'^2 \sigma_j^2$$

They proceed, erroneously, to equate λ and λ' to produce a new matrix, H, such that

$$\Delta r^T H^{-1} \Delta r = 1$$

The eigenvalues of H are the components u, v and w in each of three, orthogonal directions of the maximum position error. The maximum position error itself is clearly

$$\Delta r_{max} = \sqrt{u^2 + v^2 + w^2}$$

which does not lie within the ellipsoid the axes of which are u, v and w.

The approach of Wolff and de Wardt is therefore rejected in favour of a new method, described below, which places the extraction of ellipsoids or ellipses on a firmer statistical basis and, by establishing the values of λ and λ' directly, may

be used to extract ellipsoids and ellipses of any desired confidence level.

Unlike the method of Wolff and de Wardt, in which the measuring errors are quoted as maximum possible errors, it is assumed that they are normally distributed around a mean of zero. This assumption is expected and borne out by experimental evidence. Even if not generally provable, the combination of error distribution will, by the Central Limited Theorem, tend to produce a normal distribution for the resulting position error. Furthermore, the assumption acknowledges the fact that it is difficult to ascertain exactly what the maximum individual measuring errors are.

The two definitions, although differing, may be related. Suppose the jth measuring error is thought to lie between limits of $\pm\Delta M_j$. Since Δm_j is assumed to follow a normal distribution with mean zero it implies that $k = \Delta M_j/\sigma$ (which is equivalent to λ') is a standard normal statistic. Thus, if it is desired to establish that Δm_j lies between the desired limit upto a high degree of confidence it is simply a matter of choosing a value of k from the standard normal tables which gives a high probability. For example k = 4 gives a probability of close to 99.99%.

Now, if each individual measuring error is normal, it follows that the region of uncertainty associated with the measurement of a point will follow a 3-variate normal distribution. Suppose, more generally, that

$$x = (x_1,\ x_2, \ldots x_p)$$

is a co-ordinate in p-space, which follows a p-variate normal distribution with mean μ and covariance matrix COV, i.e. $x \sim N_p(\mu,\ COV)$, and where each individual component x_j $(1 \leq i \leq p)$ follows a normal distribution, i.e. $x_j \sim N\left(\mu_j, \sigma^2\right)$.

The multi-variate normal distribution will have constant probability densities on surface of the form:

$$(x - \mu)^T\ COV^{-1}\ (x - \mu) = \lambda^2 \qquad (5)$$

It can be shown that the U statistic

$$U = (x - \mu)^T \; COV^{-1} \; (x - \mu)$$

follows a Chi squared distribution of p degrees of freedom, i.e. $U \sim \chi_p^2$. Thus the form of a surface describing a region of uncertainty in p dimensions can be determined upto any specified confidence level using (5) where λ is given by

$$\lambda = \sqrt{\chi^2_P}$$

which in three dimensions simplifies to

$$\lambda = \sqrt{\chi^2_3}$$

Clearly, the dimensions of the ellipsoid of desired probability are therefore the square root of the eigenvalues of λ^2 COV.

The ellipse defining the boundary at a given confidence level in the horizonal (XY) and vertical (XZ and YZ) planes may be obtained by extracting the appropriate 2 by 2 matrices from COV and using the Chi square statistic for 2 degrees of freedom. Likewise the confidence level along any of the X, Y and Z axes may be determined using the Chi square statistic for one degree of freedom.

To determined the ellipse of uncertainty in an arbitrary plane, i.e. one other than XY, XZ, YZ, it is necessary to integrate the probability density function described by the covariance matrix with respect to the direction normal to the plane of interest. The ellipse thus produced is, it should be noted, neither a section nor a projection of the ellipsoid. Although such a method can be implemented, the same result can be obtained by transforming COV into a new co-ordinate system, X'Y'Z', in which one of the axes in aligned in the desired direction. The eigenvalues and eigenvectors of the matrix thus obtained may then be used in the manner described above. The validity of this approach can be seen by considering the case where the desired co-ordinate system is that of the ellipsoid

itself. In this circumstance, COV is transformed into a diagonal matrix, the leading terms of which are the variances in the directions corresponding to the axes of the ellipsoid and the eigenvectors are X′, Y′ and Z′. Since such transformations are orthogonal the eigenvalues remain, of course, those of COV.

Referring to Figure 1, points r_1 and r_2 are points along boreholes A and B respectively, which have been acquired using some borehole surveying technique.

The true position of well A, $\overline{r_1}$, will lie somewhere in the ellipsoid centred about r_1 and the true position of well B, $\overline{r_2}$, will lie somewhere in the ellipsoid centred about r_2. Note that the centres of the ellipsoids will be offset from the boreholes if any biased systematic terms, such as a drill string magnetization error, are considered in the determination of the position uncertainty. If the radii of the boreholes are R_1 and R_2 respectively then, since the direction of the well paths are not known precisely, all that can be said about the surfaces of the boreholes perpendicular to the direction of the well paths at these points is that they lie somewhere in spheres of radius R_1 centred about r_1 and R_2 centred about r_2 respectively. Hence it is certain that the boreholes will not coincide at these particular points provided $d = |\overline{r}|$, the distance between r_1 and r_2, is greater than the sum of the borehole radii.

Assuming that the individual sources of measuring error for a survey tool are approximately normally distributed and independent of one another, which assumption is borne out by practical observation, it follows by the Central Limit Theorem, that each of the coordinates at $\overline{r_1} = (\overline{x_1}, \overline{y_1}, \overline{z_1})$ will tend towards being normally distributed with expected values $\overline{x}_1$, $\overline{y}_1$, $\overline{z}_1$, and variances σ_{x1}^2, σ_{y1}^2, σ_{z1}^2, (say). Similarly each of the coordinates at $\overline{r_2} = (\overline{x}_2, \overline{y_2}, \overline{z}_2)$ will tend towards being normally distributed.

Thus, if it is assumed that the points $\overline{r_1}$ and $\overline{r_2}$ are independent of one another then by the properties of combining normal distributions it follows that the co-ordinates of the

vector $\bar{r} = (\bar{x}, \bar{y}, \bar{z})$ given by $\bar{r}_2 - \bar{r}_1$ (i.e. the position vector which joins the two points at $\bar{r}_1$ and $\bar{r}_2$) will each be normally distributed with expected values of x, y and z respectively and variances $\sigma_x^2 = \sigma_{x1}^2 + \sigma_{x2}^2$, $\sigma_y^2 = \sigma_{y1}^2 + \sigma_{y2}^2$ and $\sigma_z^2 = \sigma_{z1}^2 + \sigma_{z2}^2$

Now the expected value E(r) of the random variable

$\bar{r} = \bar{r}_2 - \bar{r}_1 = (\bar{x}, \bar{y}, \bar{z})^T$

is

$E(\bar{r}) = (E(\bar{x}), E(\bar{y}), E(\bar{z}))^T = (x_2 - x_1, y_2 - y_1, z_2 - z_1)^T$

and the variance matrix (or covariance matrix), V(r), is

$$E(\bar{r}) = E\{[\bar{r}-E(\bar{r})][\bar{r}-E(\bar{r})]^T\}$$

$$= E\{[\bar{r}_1-E(\bar{r}_1)][\bar{r}_1-E(\bar{r}_1)]^T\} + E\{[\bar{r}_2-E(\bar{r}_2)][\bar{r}_2-E(\bar{r}_2)]^T\}$$

$$\begin{matrix} \sigma_{x1}^2 + \sigma_{x2}^2 & \sigma_{x1y1} + \sigma_{x2y2} & \sigma_{x1z1} + \sigma_{x2z2} \\ \sigma_{x1y1} + \sigma_{x2y2} & \sigma_{y1}^2 + \sigma_{y2}^2 & \sigma_{y1z1} + \sigma_{y2z2} \\ \sigma_{x1z1} + \sigma_{x2z2} & \sigma_{y1z1} + \sigma_{y2z2} & \sigma_{z1}^2 + \sigma_{z2}^2 \end{matrix}$$

if $\bar{r}_1$ and $\bar{r}_2$ are independent

Thus, given that $\bar{r} = (\bar{x}, \bar{y}, \bar{z})^T$ where $\bar{x}$, $\bar{y}$ and $\bar{z}$ follow the normal distributions

$$\bar{x} \sim N(x_2 - x_1, \sigma_{x1}^2 + \sigma_{x2}^2)$$

$$\bar{y} \sim N(y_2 - y_1, \sigma_{y1}^2 + \sigma_{y2}^2)$$

$$\bar{z} \sim N(z_2 - z_1, \sigma_{z1}^2 + \sigma_{z2}^2)$$

it is assumed that $\bar{r}$ follows a 3-variate normal distribution with mean $E(\bar{r})$ and variance $V(\bar{r})$

The distribution of $d = |\bar{r}| = \sqrt{\bar{x}^2 + \bar{y}^2 + \bar{z}^2}$ (i.e. the distance between the two points) can be evaluated by taking the 3-variate normal distribution of r and making the substitution $\bar{x} = d\cos\theta\sin\phi$, $\bar{y} = d\cos\theta\cos\phi$, $\bar{z} = d\sin\theta$. Thus, it follows that the probability that the boreholes will not coincide at these points is at least as much as

P (borehole will not coincide) =

$$\int_{d=D_1+D_2}^{\infty} \int_{\theta=-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_{\phi=-\pi}^{\pi} f(d,\theta,\phi) d^2 \cos\theta \;\; d d d\theta d\phi$$

where

$$f(x,y,z) = \frac{1}{(2\pi)^{3/2} \cdot (\det V(\bar{r}))^{\frac{1}{2}}} \cdot \exp\{-\tfrac{1}{2} \cdot \bar{r}^T V(\bar{r})^{-1} \bar{r}\}$$

Referring to Figure 2, it can therefore be seen that the probability of collision can be estimated by

P (collision) = 1-P (boreholes will not coincide).

The position uncertainty at any given point along the reference well ahead of the current depth can considered to be due to contributions due to errors in the survey down to the current depth plus those which accumulate over the interval to the point of interest. The precision with which azimuth and inclination may be set and maintained is analogous to the heading reference and inclination systematic error terms of a survey tool error specification. Likewise, the circumstance considered is similar to that where a wellbore has been surveyed in two sections, each using a different survey tool. The method described above for computing the position uncertainty of such a survey is directly applicable. Note that the sources of error in the two sections are completely unrelated and so the uncorrelated form combination is the more appropriate.

One method of determining sensible values for drilling precision is by analysis of a database of expected and achieved results, suitably classified by operation, BHA type, depth, formation etc. Another is to consider the variances in azimuth and inclination observed when suitable mechanical simulations of drilling assemblies are compared against actual results. Such a process is typically used in the calibration of

mechanical simulations and is often known as "history matching".

In practical circumstances the probability of intersection of one borehole with another is of limited value, rather it is the probability of intersection with any of the boreholes beneath a drilling facility which is of interest. This may be computed according to the following method.

Let $p_1, p_2, \ldots . p_n$ be the probability of a point on the reference borehole being within a specified radius of the points of closest approach to that point of object wellbores $1 \ldots n$, i.e. the probability of intersection with each of the object wellbores. The probability of not intersecting the first object wellbore is simply $p^1{}_1 = 1-p_1$, that of not intersecting the second object wellbore $p^1{}_2 = 1 - p_2$ etc. The probability of not intersecting any of them, $P_{\text{no intersection}}$, is thus

$$P_{\text{no intersection}} = P_1{}^1 \; P_1{}^2 \ldots P^1{}_n$$

and the probability of intersection with at least one, $P_{\text{intersection}}$, is

$$P_{\text{intersection}} = 1 - P_{\text{no intersection}}$$

The method described above allows the determination of the absolute probability of intersection of a borehole with those surrounding it. For the purposes of well planning and directional control, however, it is desirable to identify positions of least risk of intersection and to navigate through those positions, such positions not necessarily being equidistant between the existing boreholes.

Considering any arbitrary point the method described can be applied by reducing its position uncertainty to nothing and reducing the tolerance around it to zero. This may be achieved assigning an empty variance matrix, i.e. a matrix with all terms zero, to the reference point and setting R_1 to zero. The probability of the closest points on each of the set of object wellbores being within the tolerance R_2 of this point may then be computed and summed as shown in the proceeding section.

By altering the value of R_1 to any appropriate number the

method described above may be used to evaluate the probability that a borehole will lie within a given tolerance of any fixed position, for instance a geological target or similar feature.

Whilst drilling it is often desirable to know the likelihood that the current reference borehole position lies along a specific direction with respect to the object borehole or, indeed, some other point such as the intended target. For example, it may be important for the reference borehole to pass beneath, or to the East or to the North, of the object well.

Consider the random variable $\tilde{z} = \tilde{z}_2 - \tilde{z}_1$ which is the Z-co-ordinate of the actual position vector between point A on the reference borehole and point B on the object borehole. If Z is taken to represent vertically downwards then $\tilde{z} > 0$ implies that the vertical depth of B is greater than that of A. Hence, if it is assumed that the vector $[\tilde{x}, \tilde{y}, \tilde{z}]^T$ follows a 3-variate normal distribution the probability that B lies below A is equivalent to the probability that $\tilde{z} > 0$ which can be determined as follows

$$P(\tilde{z}>0) = \int_{\tilde{z}=0}^{\infty} \int_{\tilde{y}=\infty}^{\infty} \int_{\tilde{x}=-\infty}^{\infty} f(\tilde{x}, \tilde{y}, \tilde{z})\, d\tilde{x} d\tilde{y} d\tilde{z}$$

Similarly it is possible to determine the probability that A is North of or East of B by considering the probability that $\tilde{y} > 0$ and $\tilde{z} > 0$ respectively.

In fact, one is not limited to this. It is also possible to determine what the likelihood of the relative position of the object well is in terms of any combination of North, East and Above. For example, it can be determined that the reference borehole is North and East of the reference borehole or even North, East and Above.

Neither is one limited in terms of North, East and Above. For example, when reference and object wellbores approach one another normally it is useful to determine the relative risks of passing to the left or right. Such a circumstance may occur when drilling an horizontal well in proximity to a vertical exploration or delineation well. By defining a new frame of reference (X', Y', Z') such that Z' is aligned with the

reference borehole vector, X' is normal to this vector in the horizontal plane and Y' is normal in the vertical plane then a matrix, M, can be derived which defines the transformation between (X', Y', Z') and (N,E,V). Applying this transformation to the variance matrix such that

$V^1 = M^T VM$

produces a transformed variance matrix which, when the method defined above is applied, will produce probabilities of "ahead / behind", "left / right" or "up / down" or any combination thereof.

A property of V^1 is that its eigenvalues will be the same as those of V although its eigen vectors will, of course, differ. Further, if the orientation of (X', Y', Z') is chosen such that they align with the eigenvectors of V then V^1 will be a diagonal matrix, the leading terms of which are the variances in the directions corresponding to the axes of the ellipsoid of uncertainty and the eigenvectors X', Y' and Z'.

Using the method described above, charts, maps, strip charts or engineering drawings showing the position of a borehole or proposed borehole, known as the "reference well", and a suitably selected group of other boreholes, such boreholes being known as "object wells", are created. Criteria for membership of the set of object wells may include, amongst others, being drilled from a specific structure, lying within a specific field or coming within a specific distance of the reference well. The drawing may display the well in either plan or vertical or isometric or other projection. The drawing may also display, instead of the actual trajectory of the boreholes, their relative separation; such charges have been called "travelling cylinder" or "rose diagram" plots. The drawing may display ellipses or ellipsoids of uncertainty at a specified level of confidence. The probability of intersection with each object well may be displayed as an attribute of the object well or the joint probability intersection with any of the object wells as an attribute of the reference well. The choice of display and the attribute employed for representation may best be decided according to the task at hand. Examples

include varying colour, weight or style of the line representing a well; using "wiggle traces" (either plain or filled) or more complex denotations such as spot sizes / colour or "tadpoles". Typically such drawings are rendered upon paper or other tangible and reproducible medium although transient displays upon a visual display unit may also be used.

The evaluation of the variance matrices and the computation therefrom of the probability of intersection and the analysis and interpretation thereof are generally performed numerically by means of a computer program. To be of practical use such a program should also encompass a means of storage and retrieval of a large number of borehole surveys, together with a means of computing the co-ordinates of stations therein (at least according to the minimum curvature method) and of interpolating the position and borehole vector of points between stations. It should also provide a facility to create and maintain associations between borehole surveys and error models. The error models should contain at least a description of the source and magnitude of the measurement errors, m_j and $m_j^{(i)}$, applicable to the type of instrumentation used to gather the data. In the circumstance where the survey is acquired in two or more sections such associations mst be maintained for each section of the survey.

In the circumstance that large numbers of survey are stored, examples being large offshore platforms or Arctic drilling locations, the program should provide a facility for the classification of boreholes and their surveys and their extraction from storage according to the classification. Such a facility simplifies the creation of sets of object wells and minimises the risk that relevant data will be inadvertently neglected.

Furthermore, the program should provide a facility for the creation of, at least, a description of the trajectory for planned boreholes in a manner analogous to that used for surveys of actual wells. It should also provide for division of the trajectory of a planned borehole into survey sections and for the association of each section with an error model.

In practice it is useful to allow the creation of many such divisions and associations in order to simulate, compare and contrast the influence of possible surveying programmes upon borehole position uncertainty at different stages of the drilling process.

To support this function it is desirable that a means is provided to specify the surface location of drill sites and the position and orientation of specific wellheads (slots) therein and the uncertainty in position of the drill site and the slots.

In addition it is necessary for such a program to provide a function for the computation of the separation of boreholes one from another in the horizontal plane, the plane normal to the borehole vector and three dimensions and the determination therefrom, by iterative or other means, of the points of closest approach of the reference well with each member of the set of objects wells.

Each of these attributes of a practical program may be implemented in a variety of ways using a variety of algorithms. They may be implemented within a single program or as a suite of programs sharing common data. In a preferred implementation the features are provided as a suite of inter-related programs sharing a common data and derived from common subroutine libraries.

In a preferred implementation the set of errors m_j and $m_j^{(i)}$ for magnetic survey tools are identified as:

1. Error in magnetic declination
2. Magnetic compass error
3. Inclinometer error
4. Misalignment with the centre line of the borehole
5. Along hole depth measurement error

The weighting function for error term (2) above accounts for the sensitivity of methods of compensation for drill string magnetic interference to uncertainty in determination of the Earth's magnetic field strength and dip angle.

For gyroscopic tools the errors are identified as:

1. Error in alignment with reference direction

2. Gyro compass error
3. Inclinometer error
4. Misalignment with the centre line of the borehole
5. Along hole depth measurement error

The weighting function for error term (2) above accounts for the time dependency on these errors in "continuous" survey tools.

For inertial tools the errors are identified as:

1. Error in alignment with reference direction
2. X, Y and Z displacement errors

The weighting function for error term (2) accounts for the time dependency of these errors.

For all tools the following errors are identified:

1. Displacement of tool from centre line of the borehole.

The errors enumerated are representative only as the method presented can account for an arbitrary set of errors.

An extended version of this preferred implementation incorporates the facility to describe drilling precision in a manner suitable for the determination of the wellbore position uncertainty.

A specific method of using a typical least risk of borehole intersection chart created according to the method presented is given below. Such a chart may be used in the initial stages of well planning to identify the safest positions to which to navigate.

1. Create base charts or maps of the location and, optionally, the associated position uncertainty of the set of object wells at specific vertical depths.

2. The area covered by the base map, or any portion thereof, is divided into cells, the shape and size of which are appropriate to the scale of the base map and appropriate to the task in hand. Each cell is aligned a value determined as the probability of borehole intersection of its centre.

3. The surface thus defined may be rendered according to any appropriate method including, but not limited to, contouring, shading or colouring.

4. By examination, either visual or otherwise (for example by use of algorithms to determine surface minima), of a set of such charts created at regular depth intervals the optimum position of each depth for a new reference well may be determined.

5. A modification of the above method is appropriate for visual displays wherein the base map of step (1) and the cells defined in step (2) are three dimensional and rendered using appropriate 3 dimensional visualisation software.

A process of drilling incorporating the new method is as follows:

1. Determine the desired course of the borehole according to the procedure described above or any other appropriate to the circumstance at hand.

2. Set forth a safe limit on the allowable risk of borehole intersection according to either the relevant health, safety or environment regulations extant at the time and place of drilling or the financial risk, which ever is the lesser.

3. Drill an initial length of the borehole, the BHA and drilling parameters (particularly weight-on-bit and rotary speed) used being selected to give adequate control for the section of well.

4. At regular intervals along the borehole take observations of the borehole inclination and azimuth using either magnetic or gyroscopic instruments and hence determine both the position of the bit and its associated position uncertainty.

5. Using a mechanical simulation of the BHA or otherwise estimate the trajectory of the borehole over the next length to be drilled and determine first the position and associated position uncertainty of the drill bit at the end of this length according to the drilling tolerances of the BHA and the quality of the estimation and thence the risk of intersection.

6. If the risk of intersection exceeds the safe limit set forth in step (2), repeat step (5) successively considering alternative trajectories achievable with the BHA in use and

select the safest trajectory compatible with reaching the desired objective and remaining within other drilling constraints, for example torque or drag limits.

7. If the safest achievable trajectory exceeds the safe limit, repeat step (5) successively considering alternative BHA configurations.

8. If the safest achievable trajectory exceeds the safe limit, repeat step (7) considering trajectories achievable from prior points along the borehole.

A preferred embodiment of this process offers advantage over prior art by permitting the computed risk of intersection to be associated with cost related data, for example the operating cost per hour of the drilling rig. Using this preferred embodiment decisions such as whether or not to extract and replace the drilling assembly or the depth at which to plug and sidetrack a borehole the trajectory of which cannot be adequately controlled, both of which are expensive operations, can be made on an objective, quantative basis; presently such decisions are made by subjective judgement which may, retrospectively, prove incorrect.

It will be appreciated by persons skilled in the art that various alterations and modifications are possible without departure from the scope of the invention as defined by the appended claims.

CLAIMS:

1. A method of determining the risk of a collision of a borehole, the method comprising:

measuring azimuth and inclination of the borehole at intervals along it using magnetic or gyroscopic instruments, or measuring the total displacement from a surface location of successive points using an inertial instrument;

determining the magnitude of systematic and random errors inherent in the measurements made and their relationship with azimuth and inclination of the borehole and displacement from the origin and elapsed time at each successive point of observation;

determining a first variance matrix representing position uncertainty at each location at which said measurements are made;

determining the point of closest approach of the borehole to a reference point;

determining second variance matrices representing the position uncertainty at said reference point and said point of closest approach;

determining a vector between the reference point and the point of closest approach;

determining a normalised distribution of the length of said vector according to the second variance matrices; and

integrating the normalised distribution between limits of zero and a value equal to the sum of a predetermined radius and the radius of the borehole, wherein the integral obtained represents the probability that the borehole lies within the predetermined radius of the reference point.

2. A method according to claim 1, wherein the step of determining the first and / or second variance matrix and / or the normalised distribution are performed by application of a stochastic technique, and preferably by application of Monte Carlo modelling, and the integrating step is performed by examination of the frequencies thus derived.

3. A method according to claim 1 or 2, wherein the first variance matrix is determined from estimates of expected

drilling precision.

4. A method according to any one of the preceding claims, wherein the reference point lies along an existing or intended borehole.

5. A method according to claim 4, wherein the integrating step further comprises setting the predetermined radius around the reference point to the radius of the borehole along which the reference point lies.

6. A method according to any one of the preceding claims, wherein the method is applied iteratively to a set of boreholes and the integrals obtained from the integrating step are summed.

7. A method according to claim 6, further comprising the step of geometrically transforming the second variance matrices, prior to the determination of the normalised distribution, into an arbitrary co-ordinate system and wherein said integrating step comprises restricting the integral to a single dimension so that it represents the probability that the borehole lies within a specific distance and direction of the reference point.

8. A method according to any one of the preceding claims, wherein the reference point is the position of the bit of a borehole drilling apparatus located within a borehole, and the arbitrary co-ordinate system is defined by the direction and inclination of the borehole at that position.

9. A method of borehole drilling comprising a method according to any one of the preceding claims, and controlling drilling of the borehole according to the probability obtained from the integrating step.

10. A method of determining position uncertainty in a directional borehole, the method comprising:

measuring the azimuth and inclination of the borehole at intervals along it using magnetic or gyroscopic instruments, or measuring the total displacement of successive points from a surface location using an inertial instrument;

determining the magnitude of systematic and random errors inherent in the measurements made;

modelling the relationship of the magnitude of both systematic and random errors with azimuth and inclination at the measuring locations, and its displacement from the origin and elapsed time as a weighting function;

determining a variance matrix representing position uncertainty of each measurement location; and

obtaining from the variance matrix an ellipsoid bounding the volume of space containing the probable position at a predetermined level of confidence of each measurement position.

11. A method according to claim 10, wherein the step of determining the variance matrix accounts for the contribution to the variance matrix of the uncertainty in the position of the surface location.

12. A method according to claim 10 or 11, wherein the step of determining the variance matrix accounts for the contribution to the variance matrix of the correlation between individual systematic error terms in successive sections of a borehole survey run in multiple sections.

13. A method of determining position uncertainty in a directional borehole, the method comprising:

measuring the azimuth and inclination of the borehole at intervals along it using magnetic or gyroscopic instruments, or measuring the total displacement of successive points from a surface location using an inertial instrument;

determining the magnitude of systematic and random errors inherent in the measurements made;

modelling the relationship of the magnitude of both systematic and random errors with azimuth and inclination at the measuring locations, and its displacement from the origin and elapsed time as a waiting function;

determining a variance matrix representing position uncertainty of each measurement location; and

obtaining from the variance matrix an ellipse defining an area containing the probable position at a predetermined level of confidence of each measurement position, or obtaining the probability in a predetermined direction.

14. A method according to any one of claims 10 to 13, further comprising the step of determining the minimum number of survey sections and measurement locations required to achieve a predetermined level of confidence by iterative application of the method.

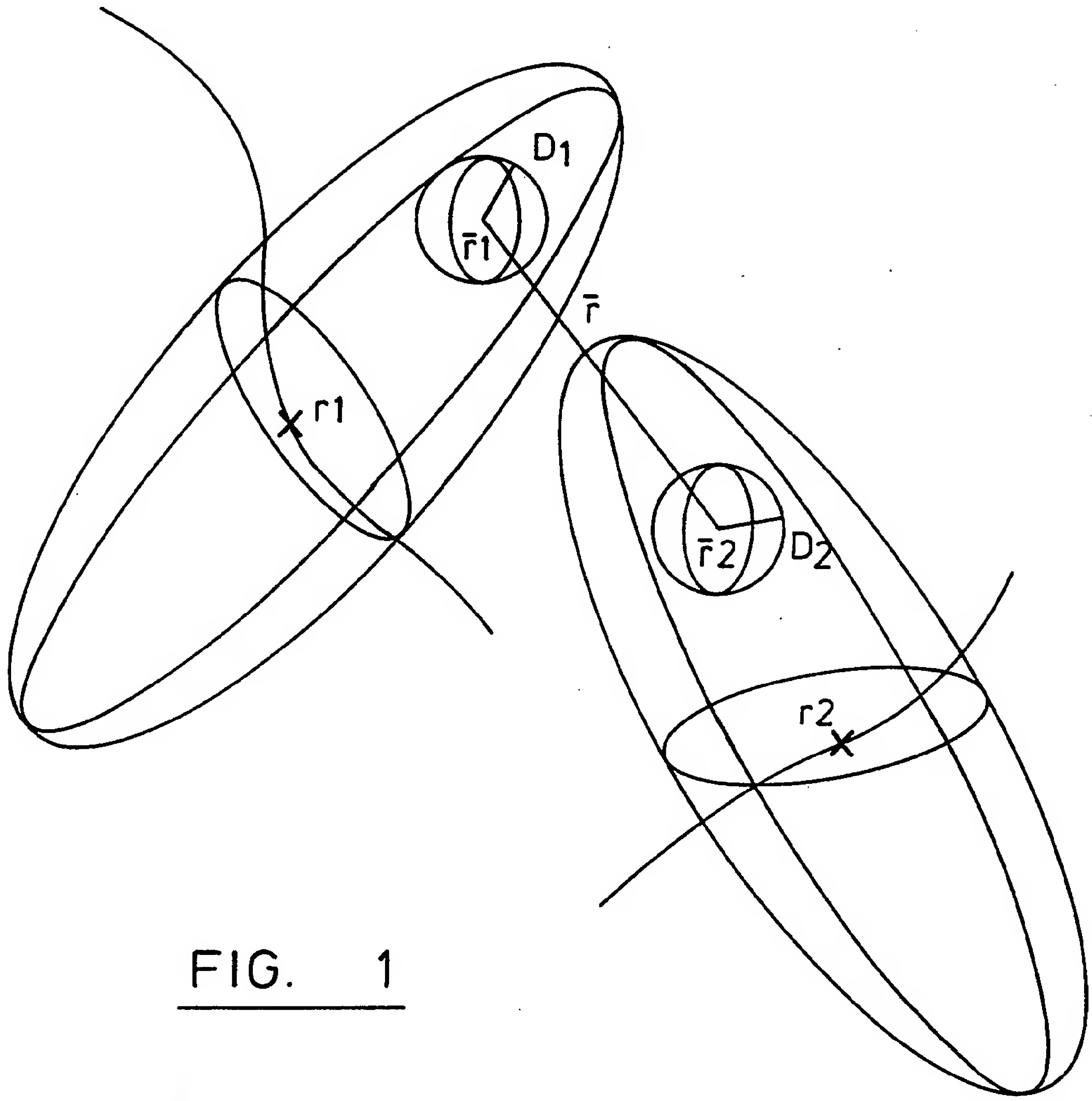

FIG. 1

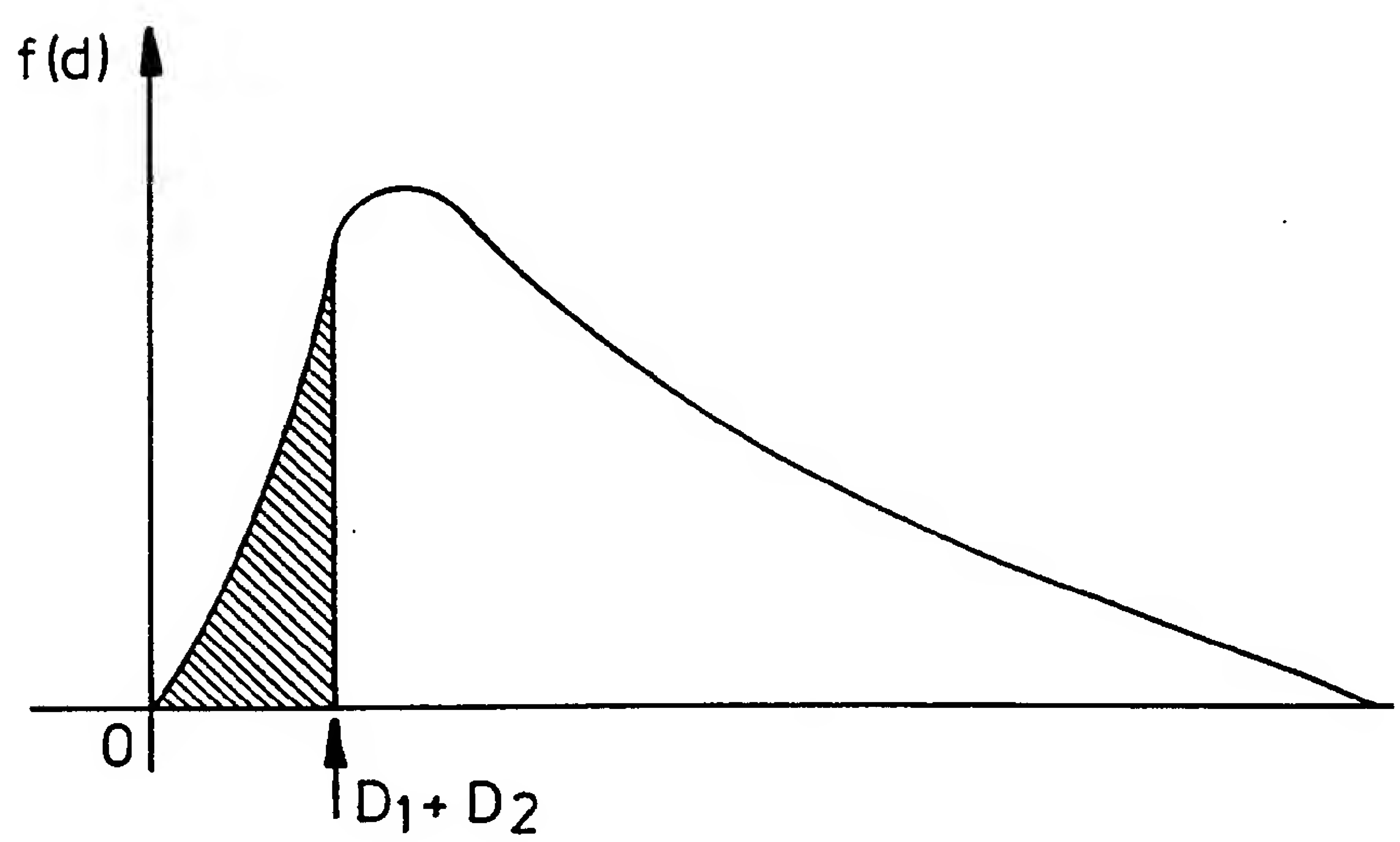

FIG. 2

INTERNATIONAL SEARCH REPORT

International Application No
PCT/GB 96/01123

A. CLASSIFICATION OF SUBJECT MATTER
IPC 6 E21B47/022
According to International Patent Classification (IPC) or to both national classification and IPC

B. FIELDS SEARCHED
Minimum documentation searched (classification system followed by classification symbols) IPC 6 E21B
Documentation searched other than minimum documentation to the extent that such documents are included in the fields searched
Electronic data base consulted during the international search (name of data base and, where practical, search terms used)

C. DOCUMENTS CONSIDERED TO BE RELEVANT

Category *	Citation of document, with indication, where appropriate, of the relevant passages	Relevant to claim No.
X	US,A,4 957 172 (PATTON BOB J ET AL) 18 September 1990 see column 5, line 53 - column 13, line 27 ---	1,10,13
A	JOURNAL OF PETROLEUM TECHNOLOGY, vol. 33, no. 7, July 1981, pages 2339-2350, XP000579591 WOLFF ET AL.: "Borehole Position Uncertainty-Analysis of Measuring Methods and Derivation of Systematic Error Model" cited in the application see the whole document --- -/--	1

[X] Further documents are listed in the continuation of box C.

[X] Patent family members are listed in annex.

* Special categories of cited documents :

"A" document defining the general state of the art which is not considered to be of particular relevance

"E" earlier document but published on or after the international filing date

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"O" document referring to an oral disclosure, use, exhibition or other means

"P" document published prior to the international filing date but later than the priority date claimed

"T" later document published after the international filing date or priority date and not in conflict with the application but cited to understand the principle or theory underlying the invention

"X" document of particular relevance; the claimed invention cannot be considered novel or cannot be considered to involve an inventive step when the document is taken alone

"Y" document of particular relevance; the claimed invention cannot be considered to involve an inventive step when the document is combined with one or more other such documents, such combination being obvious to a person skilled in the art.

"&" document member of the same patent family

Date of the actual completion of the international search	Date of mailing of the international search report
22 August 1996	13.09.96
Name and mailing address of the ISA European Patent Office, P.B. 5818 Patentlaan 2 NL - 2280 HV Rijswijk Tel. (+31-70) 340-2040, Tx. 31 651 epo nl, Fax: (+31-70) 340-3016	Authorized officer Hoekstra, F

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C.(Continuation) DOCUMENTS CONSIDERED TO BE RELEVANT

Category °	Citation of document, with indication, where appropriate, of the relevant passages	Relevant to claim No.
A	JOURNAL OF PETROLEUM TECHNOLOGY, April 1969, pages 515-523, XP000579582 JE WALSTROM ET AL.: "An Analysis of Uncertainty in Directional Surveying" cited in the application see the whole document -----	1

1

INTERNATIONAL SEARCH REPORT

Information on patent family members

Inte: onal Application No
PCT/GB 96/01123

Patent document cited in search report	Publication date	Patent family member(s)	Publication date
US-A-4957172	18-09-90	US-A- 5103920	14-04-92